Parallel Computation of Functions on Set Partitions

Abstract

Many algorithms of practical interest require evaluation of a given function \( F \) on each point of a domain consisting of all \( k \)-partitions of an \( N \)-element set. Because the cardinality of such a domain grows rapidly for fixed \( k \) and increasing \( N \), such algorithms are appealing candidates for parallelization; but to implement such parallelization efficiently in a multi-threaded (e.g., GPU/CUDA) architecture requires that each of Stirling_2(N,k) threads determine — as a function of thread index alone, in time independent of the thread index, and without recourse to inter-thread communication — a unique corresponding \( k \)-partition of the given \( N \)-element set. While a number of sequential algorithms are known for recursively enumerating all \( k \)-partitions of an \( N \)-element set, none of those algorithms can be parallelized while satisfying the requirements above, since each requires that the \( m^{th} \) \( k \)-partition in the enumeration be known before the \( (m+1)^{st} \) \( k \)-partition can be computed. This thesis project comprised the design, coding, and testing of a parallel algorithm and corresponding CUDA implementation which do satisfy those requirements.

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